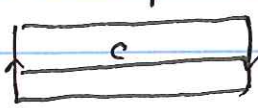
Math 74-114 Homework 1 - Sketch of Solutions

#1 1 face, 5 edges, 1 vertex $\therefore \chi(S) = -3$. S is non-orientable since it has an embedded Möbius strip. $\therefore S \approx P^{(5)}$

#2 If n is even, there is one vertex. $\therefore \chi = 2-n$. The surface is orientable since any closed curve given by a straight line from edge x to the corresponding point on x^{-1} does not reverse orientation. $\therefore S \approx T^{(\frac{n}{2})}$.

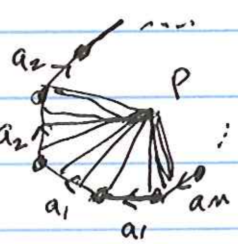
If n is odd, there are 2 vertices $\therefore \chi = 3-n$ so $S \approx T^{(\frac{n-1}{2})}$.

#3 Let $r: X \to A$ be the retraction and $x \notin A$. Then $rx \neq x$. Separate these points by disjoint open sets V and U. Then Then $V \cap r^{-1}(U \cap A)$ is a nbh of x disjoint from A. $\therefore \complement A$ (the complement of A) is open.

#4 

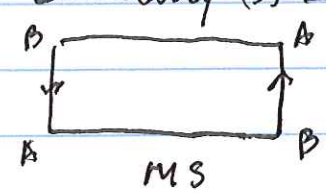
The central circle C is a sdr.

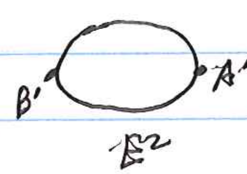
#6 Will do it for a non-orientable surface S. Let $P \in$ interior of polygon



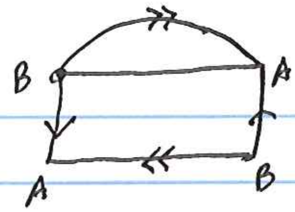
By moving a line segments from P to the boundary, we obtain a sdr of $S-P$ onto the boundary. But after identification the boundary is a wedge of n circles.

#7 X is contractible $\Leftrightarrow id_X \simeq c_{x_0}$ for some $x_0 \in X$. If X is contractible and $f: X \to Y$, then $f = f\,id_X \simeq f\,c_{x_0} = c_{f(x_0)}$. Similarly for $g: Y \to X$. Thus (1) $\Rightarrow$ (2) and (1) $\Rightarrow$ 3. If $f: X \to Y$ is nullhomotopic for any f, take $f = id_X$. Thus (2) $\Rightarrow$ (1). Similarly (3) $\Rightarrow$ (1).

#8 



Glue $A \leftrightarrow A'$, $B \leftrightarrow B'$ Getting



Then the arc BA is to be identified with AB (both indicated with $\gg$).
This gives P, projective plane.

\#9

\#10 If $hf \simeq id$ and $fg \simeq id$, then

$g = id\, g \simeq hfg \simeq h\, id = h$ $\therefore$ f is a homotopy equiv. with homotopy inverse g ($\simeq h$).

Now suppose $hf \simeq \theta$ and $fg \simeq \mu$, where θ, μ are homotopy equivalences with homotopy inverses $\bar{\theta}$ and $\bar{\mu}$.

Then $(\bar{\theta} h) f \simeq id$ and $f(\theta \bar{\mu}) \simeq id$ so f is a homotopy equivalence.

\#11 Let $i: A \to B$, $j: B \to X$ be inclusions and $r: B \to A$, $s: X \to B$ retractions. Then rs is a retraction for the inclusion $ji: A \to X$ If $js \simeq id$, $ir \simeq id$ then

$(ji)(rs) \simeq j\, id\, s = js \simeq id$.

Show if $ir \simeq id$ rel A and $js \simeq id$ rel B then $(ji)(rs) \simeq id$ rel A

\#12 Let l be a path in X from x_0 to x_1. Define $F(x_0, t) = l(t)$.

delete { \#13 Define $l': [a,b] \to X$ by
$l'(t) = l\left(\frac{t-a}{b-a}\right)$ }

\#13 Consider the dr of $\mathbb{R}^3 - 0$ onto S^2. When this is restricted to ~~X~~ $X = \mathbb{R}^3 -$ z-axis we get a dr of X onto $S^2 - \{NP, SP\}$. But $S^2 - \{NP\} \approx \mathbb{R}^2$ (stereographic projection) so $S^2 - \{NP, SP\} \approx \mathbb{R}^2 - 0$. But $S^1 \subseteq \mathbb{R}^2 - 0$ is a dr. $\therefore X \equiv S^1$